Prelaboratory Exercise 9

Objective

In this Prelab you will start the design of a lead compensator based on a given model and specific design requirements. You will implement this lead compensator during Lab 9.

References

Lecture notes: Lead Compensator & Lag Compensator

Basic Theory and Notation

A lead compensator has the form:

$$C(s) = K_C\left(\frac{Ts+1}{\alpha Ts+1}\right) \tag{1}$$

with $0 < \alpha < 1$. You can plot the Bode diagram (shown in Figure 1) to see how it contributes to the gain of the system.

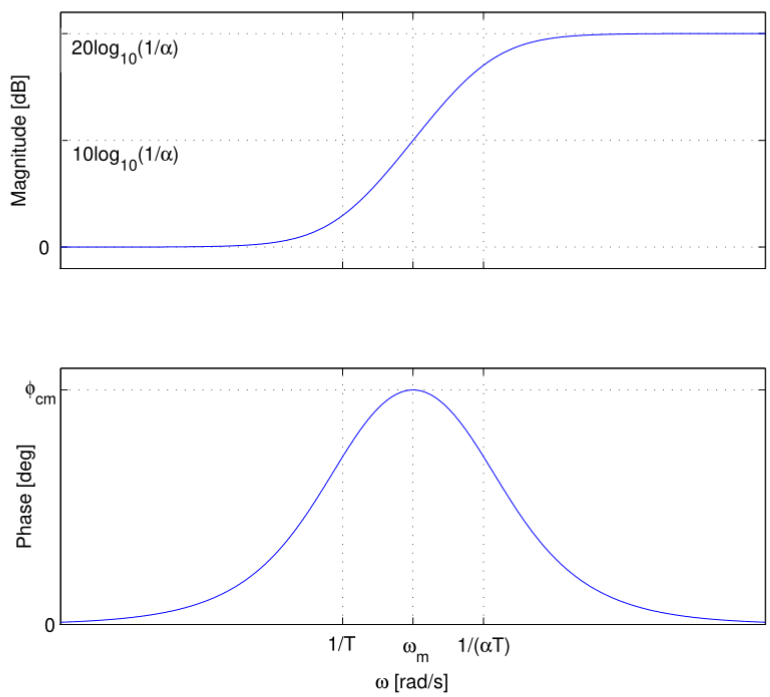

Figure 1-Bode diagram of lead compensator

In this lab, lead compensators will be the controllers and our plants will be the axes of the XY stage. The control for a single stage is shown in Figure 2.

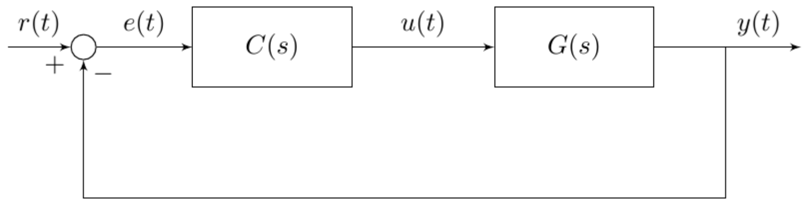

Figure 2-Closed Loop System

Overview of Design Process

Design of a lead compensator for time domain specifications typically involves the following steps:

- Obtain a plant model and design requirements (e.g. steady state error, phase margin, and gain margin).
- Determine the dc gain of the controller based on the steady state error requirement.
- Draw the Bode plot of the open loop system with the dc gain selected, then find what phase lead ϕ_{cm} needs to be added to satisfy phase margin requirement. Add a lead compensator according to the desired phase lead, ϕ_{cm} and the maximum phase contribution frequency, ω_m, using the following equations.

$$\alpha = \frac{1 - \sin\ \phi_{cm}}{1 + \sin\ \phi_{cm}}, \qquad 20\log_{10}|C(j\omega_m)G(j\omega_m)| = 0\,, \qquad T = \frac{1}{\sqrt{\alpha}\omega_m}$$

Design of a Lead Compensator

Consider a system of the form:

$$G(s) = K_{pl}\frac{\omega_{pl}}{s(s + \omega_{pl})} \tag{2}$$

where K_{pl} and ω_{pl} are the *plant's* gain and corner frequency. Use controller $C(s)$ with $G(s)$.

Question 1: As part of the design of a lead controller, you need to calculate the static error constants of the closed loop system. In this case, suppose we have a static velocity error constant specification K_v. Solve for K_c in terms of K_v and the coefficients of C(s) and G(s) using the final value theorem: $K_v = \lim_{s\to 0} sC(s)G(s)$.

Question 2: Since we want $\boldsymbol{\omega_m}$ to be our new gain crossover frequency we need , $20\log_{10}|C(j\omega_m)G(j\omega_m)| = 0$ when $T = \frac{1}{\sqrt{\alpha}\omega_m}$ and $20\log_{10}|C(j\omega_m)| = 20\log_{10} K_c + 10\log_{10}\frac{1}{\alpha}$. **Find the equation expressing the gain of the plant in decibels. Use these equations to solve for $\boldsymbol{\omega_m}$.**

Recall: $20\log_{10}\left|K_1\frac{2}{(j\omega+2)}\right| = 20\log_{10}|K_1| + 20\log_{10}|2| - 20\log_{10}|j\omega+2|$

$$= 20\log_{10} K_1 + 20\log_{10} 2 - 20\log_{10}\sqrt{\omega^2+2^2}$$

Question 3: Sketch Bode plots for both G(s) and the combination C(s)G(s). You do not have to plug in values for any variables, simply label your axes with expressions of variables as done in Figure 1.
(**Assume $\boldsymbol{\omega_m}$ is located at $\boldsymbol{\omega_{pl}}$ for this question.** During the lab, you will calculate ω_m according to your answer in Question 2.)