Prelaboratory Exercise 6

Objective

In this lab you will identify the two transfer functions of the XY stage located in the hydraulics lab. The first transfer function will be of the belt driven Y stage and the second transfer function will be of the lead-screw driven X stage.

Background

For the last three labs of the semester, you will be working with the XY stage found in the hydraulics lab. By Lab 7 you will be commanding this XY stage to follow a desired trajectory. Each stage is driven by a brushless DC motor. For the Y stage, the motor is connected first to a gear reduction and then to a belt to drive the Y motion. For the X stage, its motor is also geared and then attached to a lead-screw to create the X motion.

In order to simulate this system in the next two labs, you will need to identify the transfer functions that relate the X motion's command voltage (*proportional to force*) to millimeters traveled in the X direction and the Y motion's command voltage (*proportional to force*) to millimeters traveled in the Y direction. For this identification you will be ignoring the static friction and the coupling forces between the two axes and just identify one direction's transfer function at a time. So the identification will not be perfect, but close enough to run simulations that approximate the system. The open loop transfer function that you will identify for X axis system has the form:

$$\frac{X_{mm}}{V_{cmd}} = \frac{K_{sys}}{s(\tau_{sys}s+1)}.$$

The Y axis will have the same transfer function but with a different K_{sys} and τ_{sys}. Considering this transfer function, if you give an open-loop step voltage command to the X stage, the stage will continue to move in the X direction until it meets the end of its travel and stops. That open loop response is not too helpful in identifying the system. Also, if we apply a sinusoidal input to the system it may oscillate back and forth a few times but then drift into the limits of the travel hindering the identification of the open-loop system.

The solution to the identification of this type of system, which has position output instead of the velocity output transfer functions, is to identify a closed-loop system. The closed-loop system will keep the stage inside the limits of its travel. Then once the closed-loop system is identified the parameters of the open-loop transfer function can be solved.

The steps to identify this system is to first control the X position (or Y position) with a simple proportional controller. Produce a step response of this closed-loop system and identify the parameters, ξ *and* ω_n, of this closed-loop response. Then using the set "Kp" proportional gain you can find the closed-loop transfer function in terms of Kp, K_{sys} and τ_{sys}. Finally use the values found for ξ *and* ω_n solve for K_{sys} and τ_{sys}.

Question 1: Find the unity feedback closed loop transfer function of a proportional controller, Kp, controlling the transfer function $\frac{K_{sys}}{s(\tau_{sys}s+1)}$. Show that it is in the standard second order system transfer function form $\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$.

Question 2: Using the results you found in question 1 and given that you know Kp, solve for K_{sys} and τ_{sys} in terms of Kp, ξ *and* ω_n.

Question 3: Practice identifying ξ *and* ω_n from a step response plot. Show a plot and all your calculations. In Matlab set $\xi = 0.2$ and ω_n to 20. Then create the transfer function $\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$. Use "step" to produce a step response of this transfer function. The frequency of the decaying oscillation is called ω_d which is equal to $\omega_n\sqrt{1-\xi^2}$. Measure and verify ω_d. To find ξ you will use a method called "logarithmic decrement" which uses the fact that the peaks of the step response are exponentially decaying. Your TA will explain this more in lab, but to find ξ measure the amplitude value (distance from steady state value) of two adjacent amplitudes with amp1 being the larger amplitude and solve $\delta = \ln\left(\frac{amp1}{amp2}\right)$ and $\xi = \frac{\delta}{\sqrt{(2\pi)^2+\delta^2}}$. Also see https://www.andrew.cmu.edu/course/24-352/Handouts/logdecrement.pdf for an explanation of these equations.